Charge Superselection
Arising from Random Electromagnetic Interactions*

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The rule of superselection of charge was first stated by Wick, Wightman, Wigner[(1)] more than ten years ago; since then it has been thought of as a fundamental rule of physics. No attempt has been made, as far as we know, to explain that rule in terms of known facts; it has been only assumed as a new, independent principle.

We want to show that the superselection rule for electric charge is produced by the random electromagnetic interactions that a particle experiences in its motion through a random assembly of atoms, like in a gas. (The result is probably true for liquids or solids also but a more careful analysis is needed in these cases.)

Assume that on a particle at rest is acting a scalar potential $V(t)$. If Q is the charge operator, the potential energy is $QV(t)$, and the Schrödinger equation has the solution:

$$|t\rangle = \exp\left[-\frac{i}{\hbar}\,Q\int_0^t V(t')\,dt'\right]|0\rangle. \tag{1}$$

On account of the electromagnetic interaction, the relative phase of two eigenstates of Q, belonging to two consecutive eigenvalues, will change by

$$\delta(t) = \frac{e}{\hbar}\int_0^t V(t')\,dt' \tag{2}$$

in the time interval from 0 to t. If $V(t)$ is a random function, $\delta(t)$ will also exhibit random fluctuations, whose r.m.s. will increase as the square root of t (for t sufficiently large, and for a stationary $V(t)$). When the r.m.s. of $\delta(t)$ is somewhat larger than π, all phases are equally likely, and the pure charge state of the particle has become a mixture; i.e., a superselection rule has prevailed. The result is exactly the same as if a measurement of Q had been effected, without reading the precise value.

* *Il Nuovo Cimento*, **27** (1963), 1037.

(1) G.C. Wick, A.S. Wightman, E.P. Wigner: *Phys. Rev.* **88** (1952), 101.

As a matter of fact, we will consider a particle moving at a constant speed through a gas. In its rest reference frame, it will experience a scalar potential when entering an atom. It is not difficult to show that

$$\overline{\left(\delta(t)-\bar{\delta}\right)^2} = 8\pi\,\frac{nt}{v}\,\frac{e^2}{\hbar^2}\int_0^\infty b\,db\left(\int_0^\infty dx\,V(r)\right)^2 \qquad (t \text{ very large}), \tag{3}$$

where n is the number of atoms per unit volume, v the particle velocity, b the impact parameter with respect to the center of the atom, x the abscissa along the path of the particle from the point of closest approach to the center of the atom, $r = \sqrt{b^2 + x^2}$. $V(r)$ is the electrostatic potential of the atom, assumed spherically symmetric. (It should be noted that eq. (3) is exact even for relativistic particles, on account of the shrinking of the potential and the simultaneous dilatation of time.)

A calculation has been done, using for $V(r)$ the Thomas–Fermi potential; the resulting r.m.s. of $\delta(t)$ is greater or equal to π if

$$\frac{nlZ^{4/3}}{\beta^2} \gtrsim 2.4 \cdot 10^{20}\,\mathrm{cm}^{-2} \tag{4}$$

($l = vt$ is the distance travelled by the particle; $\beta = v/c$). If we put $\beta = 1$, $n = 3 \cdot 10^{19}\,\mathrm{cm}^{-3}$, $Z = 15$ (rather unfavourable values) we get

$$l \gtrsim 0.2\,\mathrm{cm}. \tag{5}$$

A particle travelling several millimeters in a gas (or a corresponding distance in a liquid or solid) will lose any phase relations among its charge eigenstates, and this always happens in all present-day experiments on elementary particles. The superselection rule for charge, therefore, could possibly be due only to experimental limitations.

A possible exception to the preceding considerations would be a particle produced in a cloud chamber; in the first few millimeters of its track the phase relations among different charge eigenstates would not be entirely lost, thus providing an experimental situation where the superselection of charge should not hold. It should be noted, however, that in our calculation the atom was assumed as a nucleus surrounded by a static cloud of negative charge: in that model ionization effects cannot occur whereas they are important in a cloud chamber. It is reasonable to assume that a refined model, allowing for ionization, would lead to much bigger fluctuations of the electric potential, thus reducing the "superselectionless" range to a fraction of one millimeter.

Further work is in progress on this subject.